

Branching Ratios in Proton Antiproton Annihilation at Rest from Large N_c QCD

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Abstract

We use classical or large N_c QCD to describe the mesons (π , ρ , ω) coming from proton-antiproton annihilation at rest as classical fields, which we then quantize as coherent states. This treatment gives a nearly parameter free account of the pion branching ratios in annihilation.

1 Introduction

Proton antiproton annihilation at rest goes through many channels, yielding mostly pions. Some 33% of these pions are the secondary products of annihilation first into meson resonances, principally rho and omega. The total number of pions is large. Kinematically, it can vary from two to thirteen, but the average number is about five with a variance of one. A challenge to theory is to calculate the pion spectrum and the branching ratios of the many annihilation channels. This calculation should be done in the context of the fundamental theory of the strong interactions, QCD, noting that annihilation at rest is squarely in the domain of nonperturbative QCD. In this note we exploit the large pion number to approximate the pion field as classical. This leads naturally to classical QCD, which is equivalent to QCD in the large N_C (number of colors) limit, the appropriate limit for nonperturbative QCD [1, 2]. We generate the pion, rho and omega classical fields from annihilation dynamically and then quantize the fields in the asymptotic or free field region using the method of coherent states generalized to respect isospin and four momentum conservation. From this state we obtain predictions about the branching ratios, momentum spectrum, meson number and charge type distributions. This calculation is natural in our unified treatment since we have only one state that describes the annihilation, and all the different channels emerge from projections onto that state. We find remarkable agreement with the data, in particular we correctly reproduce the trends in the branching ratios to the many annihilation channels. The only parameter we adjust is the size of the annihilation region, and that comes out at a reasonable value.

We have previously shown how large N_C QCD can be used to describe annihilation first with pions only in the context of the Skyrme model [3, 4] and then how this picture can be

extended to include the ω meson [5]. Here we extend this work to include the ρ meson as well [6] but more importantly we make an effort to connect with experiment, in particular the branching ratios [7, 8, 9]. Our previous work explored the theoretical approach of classical QCD for the dynamics, followed by quantum coherent states to impose quantum numbers and to obtain field quanta. This note establishes that approach as sensible phenomenology.

It is well known that large N_C QCD is a classical field theory of mesons only, in which baryons emerge as topologically stable nonperturbative configurations of the meson field [1, 2]. The Skyrme model [3] is the best known example of a theory incorporating these features with pions only. It begins with the non-linear sigma model, a general feature of all classical QCD pictures, and adds a fourth derivative term, called the Skyrme term, to stabilize the baryons. This added term is, probably, the first in a series of terms with more and more derivatives the exact nature of which is not known since no one has yet derived classical QCD from the quantum theory. Nevertheless, it is generally agreed that the features of any such theory at low energies will be those of the Skyrme like models, the higher derivative terms only affecting the details at higher momenta. Annihilation at rest is, in this sense, a low energy phenomenon and we believe we can therefore be led by the first few terms. It is this robust nature of Skyrme like models at low momentum that we exploit in this note. To include the ρ and ω classical fields in the large N_C treatment, we treat them as massive Yang-Mills fields which gauge the $U_V(2)$ symmetry of the non-linear σ model [10, 11]. The couplings between the vector meson fields and the pion field are fixed by the KSFR relation, [12], in terms of the vector meson mass (we take $m_\rho = m_\omega = 770$ MeV) and f_π , the pion decay constant. We follow the usual convention in Skyrme calculations and fix this constant at $f_\pi = 75$ MeV to give the nucleon mass [10]. The vector mesons stabilize the baryons, thus eliminating the need for the Skyrme term. Hence our dynamics is completely specified by only three parameters, the pion mass, the nucleon mass and the vector meson mass, all three of which we fix at their observed values.

Studies of annihilation in the Skyrme model have shown that when a Skyrmion and antiSkyrmion annihilate [13], or when a “blob” of Skyrmionic matter with zero baryon number but with energy and size appropriate to a nucleon-antinucleon pair evolves [14], pion radiation emerges as quickly as causality will permit. This picture of annihilation as proceeding in a coherent burst of classical radiation is the opposite of the traditional thermal fireball decay picture. We shall see that constructing a quantum coherent state based on the rapid burst of radiation from the classical dynamics gives an excellent account of annihilation channels. We construct this account in a simplified picture in which we begin with a spherically symmetric blob of pionic matter with total energy of two nucleon masses, and at rest. We use the dynamical equations of classical QCD to evolve the pion field and to develop the coupled omega and rho fields. From these fields in the radiation zone, we construct our quantum

coherent state, projected onto good isospin and four momentum. From this state we project the various decay channels and find the branching ratios. By starting with a blob, we skip over the difficult question of modeling the annihilation process itself and computing rates. We also cannot study Bose-Einstein correlations in this simplified picture [15]. Some compensation for these shortcomings can be found in that the blob starting point introduces only one new parameter, the size of the blob. We use this as an adjustable parameter and fit it to the pion momentum spectrum. The branching rates are then calculated with the parameters of the theory fixed by the three masses (pion, nucleon and vector meson) and by the size of the annihilation region, determined through the one pion spectrum.

2 Calculation

To begin our dynamical calculations we need an initial pion field configuration. As before [5] we parameterize that configuration as

$$F(r, t = 0) = h \frac{r}{r^2 + a^2} \exp(-r/a) \quad (1)$$

where a is a range parameter and h is fixed by the condition that the total energy be twice the proton rest mass. This form for F guarantees that the initial baryon number is zero. Note that the initial ρ and ω fields are chosen to be zero. The ρ 's and ω 's seen in annihilation are generated dynamically by their interaction with the evolving pion field. The final pion momentum distribution reflects our choice of F , but is by no means identical to it. Rather that final distribution has F passed through the non-linear dynamical equations. Our previous work was directed more at formal exposition than phenomenology, and we chose $a = 1/m_\pi$. We found that this choice of a led to a mean pion number closer to 7 than the experimental 5 and to a one body pion distribution in momentum space that peaks somewhat below the experimental value. These two discrepancies are correlated and it is clear that a smaller a will lead to higher mean momentum for the pions and correspondingly smaller average number of pions. We have carried out the dynamical calculation [6] for a range of smaller a 's and compared the resulting asymptotic pion distributions with experiment. We find that $a = 1.1$ fm gives a very good fit to the experimental momentum distribution as seen in Figure 1. It also leads to a mean pion number of 5 with variance 1, both in close agreement with experiment. We should note in passing that our simple one parameter form for the initial pion field is good enough to account for the observed pion momentum distribution. A range of 1.1 fm is also a more reasonable size for the annihilation region than a pion Compton wave length. Having fixed the size of the annihilation region by the pion momentum distribution, there are **no free parameters** left in our calculation. The other parameters are fixed by the masses.

Given the initial pion configuration we use the classical dynamical equations to generate the asymptotic π , ρ and ω fields. From them we construct the quantum coherent state corresponding to those fields. We use the projection techniques we have exploited before to define states of good four-momentum and isospin [4, 16, 17]. Since it is the total isospin we wish to specify (recall that the $\bar{p}p$ system can have either $I = 0$ or $I = 1$), and since both the ρ field and π field carry isospin, we need to project each of those fields and then combine the projected states using standard Clebsch-Gordan techniques. From all this comes a state $|I, I_z, I_1, I_2; K\rangle$ where I and I_z are the total i-spin and z-component (for $\bar{p}p$ we have $I_z = 0$) and I_1 and I_2 are the total i-spins carried by the π and ρ states. In terms of these states we can calculate the amplitude for finding a state of a fixed number of π mesons of each charge type and fixed momentum, a fixed number of ρ mesons of given charge type and momentum and a fixed number of ω mesons of given momentum. This amplitude is just the overlap of a state of the specified meson number, type, charge and momentum with the projected (normalized) coherent state. The probability, then, of finding a fixed number of π mesons of each charge type, a fixed number of ρ mesons of specified charge type and a stated number of ω mesons in a state of fixed total i-spin and z-component is the absolute square of the overlap amplitude integrated over all meson momenta and summed over all possible values of intermediate π and ρ isospin, I_1 and I_2 . To evaluate the overlaps and sums while implementing four-momentum conservation we found it necessary to use the expansion techniques developed in [4] and [5], extended to the case of three meson types.

From the calculation outlined above emerge the branching probabilities for proton antiproton annihilation at rest into each possible set of π , ρ and ω mesons separated by charge type and number. In most data compilations, only total pion numbers are quoted. It is straight forward to convert our branching ratios into the three meson types into pions only noting that each of the vector mesons has a principal decay mode into pions: $\rho^\pm \rightarrow \pi^\pm + \pi^0$, $\rho^0 \rightarrow \pi^+ + \pi^-$, $\omega \rightarrow \pi^+ + \pi^0 + \pi^-$. Armed with this formalism, we can compare our results with data.

3 Results

Let us look at the pion branching ratios. They are given in the literature as percentages. For the simpler decay modes the branching ratio is for one particular pion configuration, but decays involving many π^0 s are lumped together. In Table 1 we show the experimental branching percentages compared with our calculations. To compare with the data we need to make an assumption about isopin. We make the simple assumption of equal amounts of $I = 0$ and $I = 1$. Our results would change very little if we took the mix of 63% $I = 0$ and 37% $I = 1$ suggested by [18]. The fit to the data in Table 1 is striking. The small channels come

out small and for the large branching ratios, we are in fair, sometimes excellent agreement with experiment. In fact given the simple and parameter free nature of our treatment the agreement is surprisingly good. Our most serious relative discrepancies are for the channels, $\pi^+\pi^-$ and $\pi^+\pi^-\pi^0$. These are both two body channels since $\pi^+\pi^-\pi^0$ is dominated by $\pi\rho$. We would expect quantum corrections to our classical treatment to be largest for these channels involving the fewest quanta.

Also shown in the table is the percentage of secondary pions, that is the percentage of pions coming from resonance decays. These are nearly all from ρ and ω decay. The experimental number is 33% and we find about 30%, again in very reasonable agreement. Without the dynamical generation of vector mesons, we could not make contact with this result.

4 Conclusions

We have seen that a dynamical picture based on classical or large N_C QCD gives a remarkably accurate account of the branching ratios to the various pion and vector meson channels in $\bar{p}p$ annihilation at rest. It does this in the context of a very simple starting assumption and only one parameter. The principal new feature of our treatment is that all annihilation channels are described in terms of one single coherent state. This unified view is essential in setting the relative scale of the channels. It might be argued that our agreement with experiment simply reflects phase space that is implemented by imposing energy-momentum conservation on the quantum coherent state. Phase space is no doubt important particularly in distinguishing the large from the small channels. However, it is by no means the whole story since the vector meson fields are developed dynamically in our picture. Without those vector mesons we would not come close to fitting the data. We have done a “pions only” calculation fit to the pion spectrum, and find a far less good description of the channels [6]. Thus some aspects of the QCD dynamics are essential to our picture.

What we have done is certainly only the beginning of a complete treatment based on the approach of solving the difficult dynamics using classical QCD and quantizing afterwards to make contact with experiment. Much more remains to be done in applying the full power of this method to annihilation. For example we need to work to obtain detailed agreement with the branching ratios. This would require better treatment of quantum corrections. We should also study annihilation in flight and take account the two center nature of the process both to obtain rates and to study Bose-Einstein correlations [15]. To account for the channels with strange particles (about 7% experimentally), we need to consider large N_c QCD with $SU(3)$ flavor symmetry. We are beginning such studies. Further afield we note the remarkable correlations found in annihilation from polarized protons between the proton spin direction and the charge of pions seen in the rare two pion annihilation mode [19]. Such correlations

are difficult to explain in most pictures. However spatial-isospin correlations are a natural feature of the nonperturbative baryons generated in large N_C QCD. We are trying to extract the experimentally observed correlations from our treatment. Applications of classical QCD to processes involving many pions has been suggested in a number of other contexts, including the disoriented chiral condensate [20] and very high energy heavy ion collisions. In this later case, the spin-charge correlations of classical QCD have also been noted [21].

In summary we have shown that starting from the dynamics of classical QCD we can construct quantum coherent states that account for the principal features of proton antiproton annihilation at rest including the branching ratios to nearly all channels, large and small. The main feature of our treatment is that it treats all these channels in terms of a single quantum state and that that state is constructed from dynamical information obtained from classical QCD. Given the single pion momentum distribution from annihilation, there are no free parameters in our treatment.

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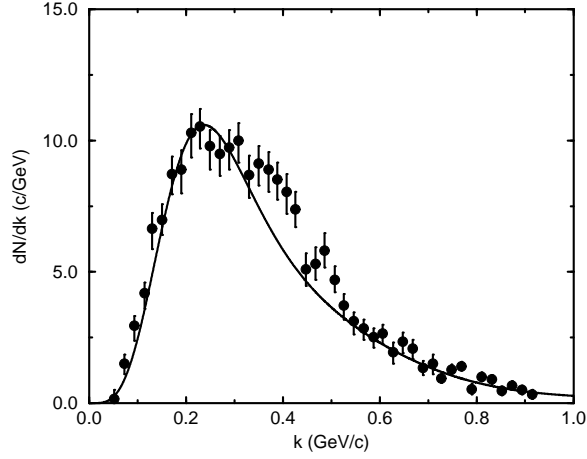


Figure 1: The inclusive single pion momentum spectrum from proton antiproton annihilation at rest. The solid curve is our calculation while the data are from [9].

Channel	Theory			Experiment	
	$I = 0$	$I = 1$	Combined	CERN	BNL
$\pi^+\pi^-$	0.02	0.0	0.01	0.37 ± 0.3	0.32 ± 0.04
$\pi^+\pi^-\pi^0$	0.04	0.6	0.32	6.9 ± 0.35	7.3 ± 0.9
$2\pi^+2\pi^-$	9.1	3.0	6.1	6.9 ± 0.6	5.8 ± 0.3
$2\pi^+2\pi^-\pi^0$	26.8	19.8	23.3	19.6 ± 0.7	18.7 ± 0.9
$3\pi^+3\pi^-$	13.8	3.56	8.7	2.1 ± 0.2	1.9 ± 0.2
$3\pi^+3\pi^-\pi^0$	4.38	0.61	2.5	1.9 ± 0.2	1.6 ± 0.2
$n\pi^0, n > 1$	7.7	15.7	11.7	4.1 ± 0.4	3.3 ± 0.2
$\pi^+\pi^-n\pi^0, n > 1$	25.1	39.8	32.5	35.8 ± 0.8	34.5 ± 1.2
$2\pi^+2\pi^-n\pi^0, n > 1$	12.8	17.4	15.2	20.8 ± 0.7	21.3 ± 1.1
$3\pi^+3\pi^-n\pi^0, n > 1$	0.03	0.014	0.022	0.3 ± 0.1	0.3 ± 0.1
% of secondary π s	29.2	31.3	30.3	33	

Table 1: Branching ratios, in percent, for proton antiproton annihilation at rest. Our calculations are compared with experiments from [7]. We show each total isospin channel calculated separately. The “combined” column corresponds to equal mixture of $I = 0$ and $I = 1$. In the last row we list the percentage of pions from the decay of rho and omega mesons.